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# An Overview of the Theory of Wire Media

C. A. Moses and N. Engheta

University of Pennsylvania  
Moore School of Electrical Engineering  
Philadelphia, Pennsylvania 19104, U.S.A.  
Tel: +1-215-898-9777, Fax: +1-215-573-2068  
E-mail: [engheta@ee.upenn.edu](mailto:engheta@ee.upenn.edu)  
Web: <http://www.ee.upenn.edu/~engheta/>

## Abstract

We present an overview of the electromagnetic theory of wire media. One can conceptually envision this class of media as composite media composed of thin wire inclusions that need not be necessarily electrically short, but can be from a fraction of a wavelength in length to multiple wavelengths in length. One can suggest several different forms and geometries for these wire inclusions; as one example of these media, we have considered the case of a medium that could be synthesized by embedding many identical, finite-length (with some arbitrary identical length), parallel, thin wire inclusions within an otherwise isotropic host medium. In this talk, we review the modeling and analysis of electromagnetic wave propagation in such media, present some results of such analysis, discuss their salient features and physical justifications, and mention some other novel inclusion geometries and shapes for wire media.

## 1. Introduction

The investigation of wave interaction with complex electromagnetic materials has been a subject of great interest over the past several years. Such interest has arisen from various perspectives—from mathematical, analytical, and computational techniques in treating radiation, scattering, and guidance of electromagnetic waves in complex media, to suggestions for potential applications, and to ideas for fabrication of some types of complex media, especially as connected to experimental verifications of their electromagnetic properties. Application of novel electromagnetic complex materials in future devices may provide new opportunities to solve some of the challenges in various fields such as wireless personal communication systems and mobile services.

Traditionally, analysis of the electromagnetic features of complex media has been more focused on realizations where the dispersed inclusions have been assumed to be small compared to the operating wavelength. However, can one suggest novel geometries for material inclusions such that while they would not necessarily be electrically short the entire ensemble may still be macroscopically considered as a *medium*? One possible scenario would be to imagine a class of artificial materials in which the inclusions are permitted to be electrically long in one dimension and yet still electrically small in the other two dimensions (i.e., the inclusions have small transverse cross sections that might vary along their length). Such inclusions can then all be positioned in parallel and in close proximity to each other so the entire structure may be macroscopically regarded as a *medium*. How then do we analyze and model electromagnetic wave propagation in such complex media?

## 2. Modeling

As an illustrative case, we consider one conceptualization of these media in which the wire inclusions are taken to be identical, but arbitrary finite-length, parallel, thin wire inclusions within an otherwise isotropic host medium. This case is an example of a larger class of complex media in which the inclusions are permitted to be electrically long in one dimension but still electrically small in the other two dimensions (e.g., the feedforward-feedbackward (FFFB) medium [1]). Furthermore, for the sake of mathematical simplicity in our analysis, we assume that these wire inclusions have been positioned on a 3-D periodic structure. However, since we want to allow the wire inclusions to have arbitrary (but identical) electrical length and at the same time we want to keep the spatial periodicities of the structure small (so an ensemble of such wires would form a macroscopic *medium*), the wire axes should be tilted away from the lattice axis.

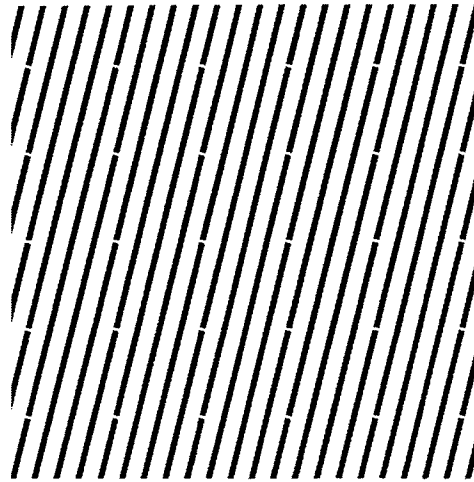
The theoretical approach we use here considers the medium as concatenation of many closely-spaced elementary planes, each of which contains a distribution of parallel identical wire inclusions located on a 2-D periodic lattice. (The types of surfaces resembling these elementary planes have been called Super-Dense dipole surfaces or Gangbuster surfaces in the context of frequency-selective surfaces (FSS) by Schneider and Munk [2], Larson and Munk [3], Kornbau [4].) The interaction of electromagnetic waves with the elementary planes is studied numerically using the standard periodic method of moments (MoM), and then, from knowledge of the wave interaction with a single elementary plane, periodic-structure theory is used to analyze wave propagation within the entire medium. Our theoretical studies have shown some interesting features connected to plane wave propagation in these media. Here we present some sample results. More results and details of our analysis of wire media are reported in [5, 6].

## 3. Reflection Properties of a Single Plane

Consider, as an example, an elementary plane where parallel identical wire inclusions are positioned on a 2-D periodic lattice as shown in Fig. 1. The periodicities of the square lattice structure is chosen as  $D = 0.13\Lambda$  where  $\Lambda$  is some reference length (not the wavelength) that may be selected to scale this surface to any desired physical size. The length and radius of these identical wires are selected as  $L = 0.533\Lambda$  and  $a = 0.001\Lambda$ , respectively. The "tilt angle" for this example is  $\alpha = 14^\circ$ . The operating wavelength  $\lambda_o$  can be chosen from a fraction of the reference length  $\Lambda$  to multiples of  $\Lambda$ .

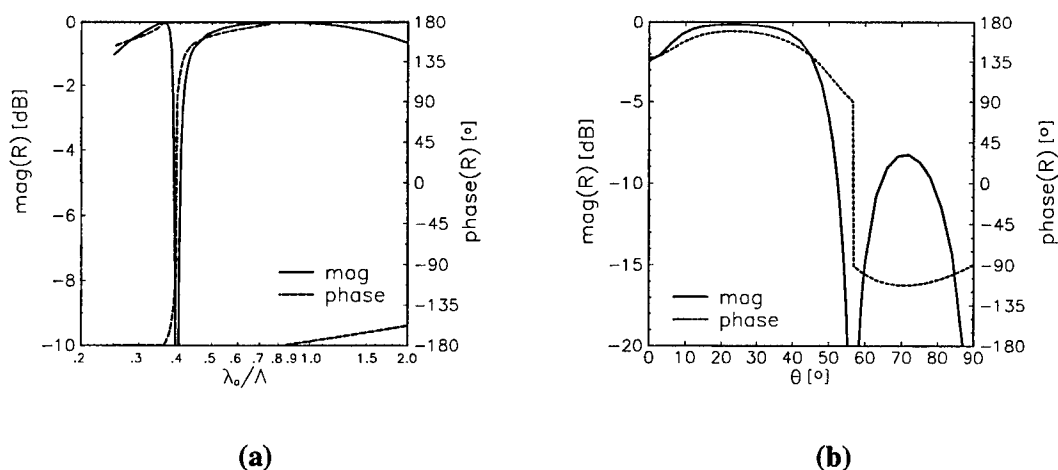
After going through a series of mathematical steps and MoM analyses [5, 6], one can find the reflection coefficient of this elementary plane when illuminated by an incident plane wave. In our investigation, we noticed that the variation of the reflection coefficient was greater as a function of angle of incidence in the E-plane (i.e., where the plane of incidence is parallel with the wire inclusions) than in the H-plane (where the plane of incidence is perpendicular to the wire inclusions) [5], so here, in the interest of space, we only present the results for the E-plane. More results can be found in [5].

Furthermore, we only consider  $TM^z$  waves (where the z-axis is parallel with the wire axes), because for these waves the incident electric field interacts appreciably with the thin wire inclusions. For the  $TE^z$  waves the thin wires do not interact with the incident wave since the incident electric field is perpendicular to the wires (the wire medium is effectively transparent to  $TE^z$  waves). Figure 2a shows the reflection coefficient of this elementary plane as a function of  $\lambda_o / \Lambda$  for a normally incident  $TM^z$  wave in the E-plane [5, 6]. At the smallest relative wavelength in this figure,  $\lambda_o / \Lambda = 0.25$ , the wires are just over two wavelengths long (i.e.,  $L = 2.13\lambda_o$ ) and at the largest relative wavelength in this figure,  $\lambda_o / \Lambda = 2$ , the wires are about  $L = 0.27\lambda_o$ .



**Fig. 1** An example of an elementary plane of a wire medium. The radius of these wires is taken to be very thin in our analysis. However, in this illustration, the wire radius is exaggerated five times in order to make it visible in print. See the text for the geometrical parameters for this example.

Note that grating lobes are not present for normal incidence throughout this range of relative wavelengths because the periodicities of the structure are chosen short enough to prevent this effect. Some of the interesting features of this figure, which are expected for the FSS, should be mentioned. (1) Two points on the reflection curve reach the point of complete reflection, i.e.,  $R = -1$ . The first point occurs at  $\lambda_o / \Lambda = 0.83$  (which indicates that  $L = 0.64\lambda_o$ ), and the second point at  $\lambda_o / \Lambda = 0.34$  (i.e.,  $L = 1.57\lambda_o$ ); (2) a deep null occurs near the point  $\lambda_o / \Lambda = 0.39$  (i.e.,  $L = 1.37\lambda_o$ ), and this is between the two wavelengths for complete reflection.



**Fig. 2** (a) Reflection coefficient of the example of single elementary plane (in Fig. 1) for a normally incident  $TM^z$  wave, as a function of relative wavelength,  $\lambda_o / \Lambda$ ; (b) the E-plane specular plane wave reflection coefficient as a function of incidence angle for the same elementary plane, with the wavelength fixed at  $\lambda_o = 0.41\Lambda$ .

This phenomenon is known in the literature as a modal interaction null [3]. Figure 2b shows the reflection coefficient as a function of incidence angle in the E-plane when the wavelength of the  $TM^z$  incident wave is kept fixed (here at  $\lambda_0 / \Lambda = 0.41$ ). As expected, the reflection coefficient is large for angles near normal incidence, and vanishes near  $\theta = 57^\circ$  and again at grazing (i.e.,  $\theta = 90^\circ$ ).

#### 4. Wave Propagation in a Wire Medium

To extend the above example to a medium, the elementary surface described above is used as the elementary plane of a wire medium. In our study, these planes are assumed to be separated far enough apart that the evanescent waves of neighboring planes do not significantly interact, but close enough that the period is small compared to the wavelength. Taking into account these points, the interplanar spacing for the example here is chosen to be at  $D_x = \Lambda / 15$  [5]. We then model the medium as a periodically loaded transmission line with the elementary planes accounted for by appropriate equivalent sheet admittances. By applying the Floquet theorem to the analysis of the periodically loaded transmission line [7], we find the propagation constant and the effective transverse impedance at the midpoint between any two adjacent elementary planes [5, 6].

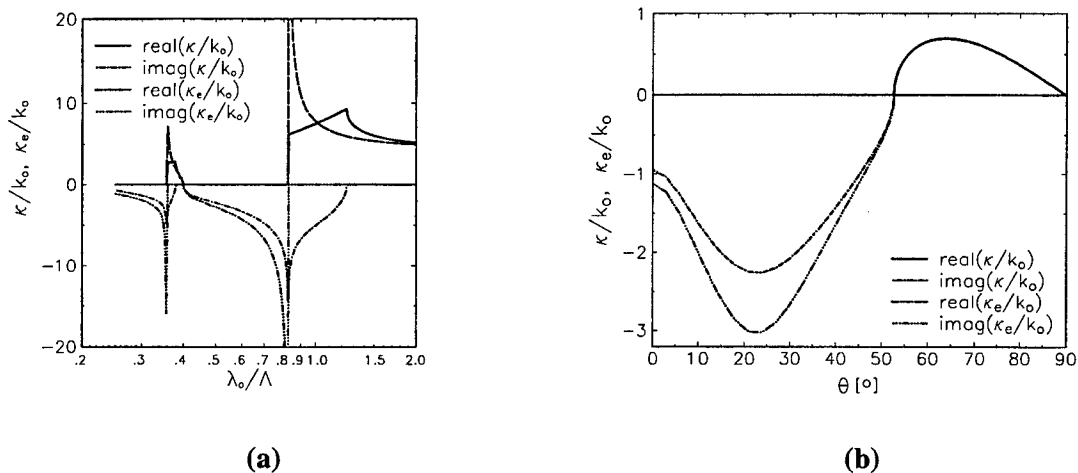
To illustrate some interesting characteristics of wave propagation in this example wire medium, consider the case where a  $TM^z$  plane wave is illuminating the interface of a semi-infinite slab of this wire medium. The wire inclusions are taken to be all parallel with the z-axis. The electric field vector of the incident plane wave is in the x-z plane. (The x-axis is normal to the interface of the medium.) One of the interesting quantities to analyze is the propagation constant inside the wire medium, particularly its component normal to the interface, denoted by  $\kappa$ . Owing to the phase-matching requirements, the tangential components (i.e., the y and z components) of the vector wave number inside the medium are the same as those components of the incident vector wave number. Figure 3a shows the normalized x-component of wave vector inside this medium as a function of relative wavelength  $\lambda_0 / \Lambda$ , for a normally incident plane wave when the interplanar spacing is taken to be  $D_x = \Lambda / 15$ .

In addition, this figure shows the corresponding x-component of wavenumber as determined by the effective media theory [5, 6]. We see from this figure that the normalized x-component of the wave vector in this sample wire medium is purely real for  $\lambda_0 / \Lambda > 1.3$ . In this region, the equivalent sheet reactance for each elementary plane is large and negative. As the wavelength decreases and reaches the range  $0.8 < \lambda_0 / \Lambda < 1.3$ , the x-component of wavenumber becomes complex. In this region, the equivalent sheet reactance is small and negative, but the wave is experiencing the bandgap effect of periodic media, mainly due to the change in the equivalent sheet reactance. In such regions, there is significant coupling between the traveling wave and the backward wave and so the energy of the forward wave is mainly transferred into the backward wave.

For this reason, the effective-media approximations are not valid in these regions (as can be seen from Fig. 3a) and, as is done here, the full periodic-media approach must be utilized. As the wavelength decreases further,  $\kappa$  is eventually purely imaginary for  $0.4 < \lambda_0 / \Lambda < 0.8$ . In this region, the equivalent sheet reactance becomes positive, and hence it represents inductive loading. Here, the wave evanescence is not due to the band-gap phenomena, but instead it is due to the heavy inductive loading of the medium. Both effective-media theory and periodic-structure theory are applicable for this region. As the wavelength continues to become shorter, one may speculate that this progression repeats, and also that the bandgap effects of the periodic lattice may appear. Figure 3b presents the normalized x-component of vector wavenumber inside this medium in the E-plane, as a function of angle of incidence of the  $TM^z$  plane wave when the wavelength is kept at  $\lambda_0 / \Lambda = 0.41$ .

In this case, the interior wave in the E-plane is evanescent for the incidence angles in the E-plane between  $\theta = 0^\circ$  and  $\theta \approx 52^\circ$ . However, for the incidence angles greater than  $\theta \approx 52^\circ$ , the wave can penetrate into the wire medium. Thus, it is evident that there is exclusion of wave propagation from certain angular regions in such a medium; essentially, wave propagation is restricted to certain "angular windows" that depend upon the chosen design parameters (e.g., number density, orientation, length, and volume fraction of inclusions).

Furthermore, these media, as expected, are frequency dependent and polarization dependent. In other words, wire media effectively behave as angle-selective, frequency-selective, and polarization-selective media. There are several other interesting features, which we obtained through our analysis of this sample, and which are mentioned in [5, 6].



**Fig. 3** (a) Normalized x-component of vector wave number,  $\kappa/k_0$ , obtained from the periodic-structure theory and its effective-media counterpart,  $\kappa_e/k_0$ , inside our example of wire media, as a function of relative wavelength,  $\lambda_0/\Lambda$  for a normally incident wave. The interplanar spacing between the elementary surfaces is taken to be  $D_x = \Lambda/15$ ; (b) The quantities  $\kappa/k_0$  and  $\kappa_e/k_0$  in the E-plane as a function of angle of incidence of a  $TM^z$  plane wave in the E-plane, at the wavelength  $\lambda_0/\Lambda = 0.41$ .

## 5. Summary

The conceptualization of wire media presents an idea for a class of complex media in which the inclusions may, under certain constraints, be chosen to be electrically long. We have presented some of the results of our analysis for a case of wire media in which inclusions are taken to be identical, arbitrary finite-length, parallel, thin wires. It is important to note that even though there seems to be many wire inclusions on each elementary plane, the volume fraction is still very small, since the wires are taken to be very thin. For instance, for the example wire medium discussed here, the volume fraction is just less than 0.15%. Despite the small volume fraction of the metal inclusions, the resulting medium may have significantly different electromagnetic properties than the host medium.

If one were to compare two blocks of dielectric—one with wire inclusions and the other without—substantial differences in the electromagnetic properties would be noticed, even though they may have almost identical weight. Owing to the interesting features of electromagnetic wave propagation in

such media, wire media, or media similar to wire media, may find some potential applications in the design of future microwave devices and components; examples include substrates for micromachined and miniature antennas, radome for beam shaping, and waveguides for mode selection.

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